## Giant resonances in ${ }^{208} \mathbf{P b}$ and the nuclear matter equation of state

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The equation of state (EOS) of nuclear Matter (NM) is an important ingredient in the study of properties of nuclei at and away from stability, the study of structure and evolution of compact astrophysical objects, such as neutron stars and core-collapse supernovae, and the study of heavy-ion collisions (HIC). In the vicinity of the saturation density $\rho_{0}$ of symmetric NM, the EOS can be approximated by

$$
\begin{equation*}
\mathrm{E}_{0}[\rho]=\mathrm{E}\left[\rho_{0}\right]+\frac{1}{18} \mathrm{~K}_{\mathrm{NM}}\left(\frac{\rho-\rho_{0}}{\rho_{0}}\right)^{2}, \tag{1}
\end{equation*}
$$

where $\mathrm{E}_{0}[\rho]$ is the binding energy per nucleon and $\mathrm{K}_{\mathrm{NM}}$ is the incompressibility coefficient which is directly related to the curvature of the EOS, $\mathrm{K}_{\mathrm{NM}}=\left.9 \rho_{0}^{2} \frac{\partial^{2} \mathrm{E}_{0}}{\partial \rho^{2}}\right|_{\rho_{0}}$. Similarly, the EOS of asymmetric NM, with proton density $\rho_{\mathrm{p}}$ and neutron density $\rho_{\mathrm{n}}$, can be approximated by

$$
\begin{equation*}
\mathrm{E}\left[\rho_{\mathrm{p}}, \rho_{\mathrm{n}}\right]=\mathrm{E}_{0}[\rho]+\mathrm{E}_{\text {sym }}[\rho]\left(\frac{\rho_{\mathrm{n}}-\rho_{\mathrm{p}}}{\rho}\right)^{2}, \tag{2}
\end{equation*}
$$

where $E_{\text {sym }}[\rho]$ is the symmetry energy at matter density $\rho$, approximated as

$$
\begin{equation*}
E_{\text {sym }}[\rho]=J+\frac{1}{3} L\left(\frac{\rho-\rho_{0}}{\rho_{0}}\right)+\frac{1}{18} K_{\text {sym }}\left(\frac{\rho-\rho_{0}}{\rho_{0}}\right)^{2}, \tag{3}
\end{equation*}
$$

where $\mathrm{J}=\mathrm{E}_{\text {sym }}\left[\rho_{0}\right]$ is the symmetry energy at saturation density $\rho_{0}$, and $\mathrm{L}=\left.3 \rho_{0} \frac{\partial \mathrm{E}_{\text {sym }}}{\partial \rho}\right|_{\rho_{0}}$, and $\mathrm{K}_{\text {sym }}=\left.9 \rho_{0} \frac{\partial^{2} \mathrm{E}_{\text {sym }}}{\partial \rho^{2}}\right|_{\rho_{0}}$.

The saturation point of the equation of state (EOS) for symmetric ( $N=Z$ ) nuclear matter (NM) is well determined from the measured binding energies and central matter densities of nuclei, by extrapolation to infinite NM. To extend our knowledge of the EOS beyond the saturation point of symmetric $N M$, an accurate value of the NM incompressibility coefficient $\mathrm{K}_{\mathrm{NM}}$, which is directly related to the curvature of the EOS of symmetric NM, is needed. An accurate knowledge of the dependence of the symmetry energy, $\mathrm{E}_{\text {sym }}(\rho)$, on the matter density $\rho$ is needed for the EOS of asymmetric NM.

There have been many attempts over the years to determine $\mathrm{K}_{\mathrm{NM}}$ and $\mathrm{E}_{\text {sym }}(\rho)$ by considering physical quantities which are sensitive to the values of $\mathrm{K}_{\mathrm{NM}}$ and $\mathrm{E}_{\text {sym }}(\rho)$. In this work we investigate the sensitivity of the strength function distributions of the isoscalar ( $\mathrm{T}=0$ ) and isovector ( $\mathrm{T}=1$ ) giant resonances with multipolarities $L=0-3$ of ${ }^{208} \mathrm{~Pb}$ to bulk properties of NM , such as $\mathrm{K}_{\mathrm{NM}}, \mathrm{E}_{\text {sym }}$ and the effective mass $\mathrm{m}^{*}$. For this purpose, we have carried out fully self-consistent Hartree-Fock (HF) based RPA calculations of the strength functions of these resonances, for ${ }^{208} \mathrm{~Pb}$, using a wide range of 34
commonly employed Skyrme type interactions. These interactions, which were fitted to ground state properties of nuclei are associated with a wide range of nuclear matter properties such as incompressibility coefficient $\mathrm{K}_{\mathrm{NM}}=200-258 \mathrm{MeV}$, symmetry energy $\mathrm{J}=27-37 \mathrm{MeV}$ and effective mass $\mathrm{m}^{*}=0.56-1.00$. We investigate the sensitivity of $\mathrm{E}_{\mathrm{CEN}}$ to physical quantities of nuclear matter $(N M)$, such as the effective mass $\mathrm{m}^{*}$; the incompressibility coefficient $K_{N M}=\left.9 \rho_{0}^{2} \frac{d^{2}(E / A)}{d \rho^{2}}\right|_{\rho_{0}}$; the skewness coefficient which is proportional to the third derivative $Q=\left.27 \rho_{0}^{3} \frac{d^{3}(E / A)}{d \rho^{3}}\right|_{\rho_{0}}$; the symmetry energy coefficient $\mathrm{J}=\mathrm{E}_{\text {sym }}\left(\rho_{0}\right)$; the symmetry energy at $0.1 \mathrm{fm}^{-3}, \mathrm{~J}(0.1)=\mathrm{E}_{\text {sym }}(0.1)$ the coefficient proportional to the slope, $L=\left.3 \rho_{0} \frac{d\left(E_{s y m}\right)}{d \rho}\right|_{\rho_{0}}$, and the curvature $K_{s y m}=\left.9 \rho_{0}^{2} \frac{d^{2}\left(E_{s y m}\right)}{d \rho^{2}}\right|_{\rho_{0}}$ of the density dependence of the symmetry energy, respectively, $\kappa$, the enhancement factor of the energy weighted sum rule for the IVGDR, and the strength of the spin-orbit interaction $\mathrm{W}_{0}$.

In Table I we present the Pearson Correlation Coefficients between the values of $\mathrm{E}_{\text {CEN }}$ of the isoscalar and isovector giant resonances, respectively, and the properties of NM. It is seen from the Table that strong correlations were found between the energies of compression mode: ISGMR and $\mathrm{K}_{\mathrm{NM}}$, and between the energies of the ISGDR, ISGQR, and ISGOR and $\mathrm{m}^{*}$. Strong correlations are found between Q and all the isoscalar modes However, weak correlations were found between the IVGDR in ${ }^{208} \mathrm{~Pb}$ and the values of $\mathrm{J}, \mathrm{L}$, or $\mathrm{K}_{\text {sym. }}$.

Table I. Pearson's correlation coefficient between the energies of Isoscalar ( $T=0$ ) and Isovector ( $T=1$ ) giant multipole ( $L=0-3$ ) resonances in ${ }^{208} \mathrm{~Pb}$ and NM properties.

|  | $\mathrm{m}^{*} / \mathrm{m}$ | $\mathrm{K}_{\text {NM }}$ | $\mathrm{K}^{\prime}=-\mathrm{Q}$ | J | $\mathrm{J}(0.1)$ | L | $\mathrm{K}_{\text {sym }}$ | K | $\mathrm{W}_{0}\left(\mathrm{x}_{\mathrm{w}}=1\right)$ |
| :--- | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | ---: |
| L0 T0 | -0.52 | 0.90 | -0.84 | -0.06 | -0.29 | 0.28 | 0.53 | -0.01 | 0.29 |
| L1 T0 | -0.91 | 0.53 | -0.85 | -0.11 | -0.11 | 0.06 | 0.36 | 0.47 | 0.20 |
| L2 T0 | -0.95 | 0.39 | -0.84 | -0.12 | -0.25 | 0.13 | 0.43 | 0.55 | 0.31 |
| L3 T0 | -0.95 | 0.29 | -0.77 | -0.02 | -0.14 | 0.13 | 0.39 | 0.59 | 0.12 |
| L0 T1 | -0.64 | 0.05 | -0.43 | -0.24 | 0.03 | -0.26 | -0.17 | 0.89 | -0.04 |
| L1 T1 | -0.53 | -0.11 | -0.26 | -0.29 | 0.19 | -0.44 | -0.38 | 0.85 | -0.05 |
| L2 T1 | -0.71 | 0.02 | -0.45 | -0.30 | 0.06 | -0.34 | -0.20 | 0.87 | 0.03 |
| L3 T1 | -0.82 | 0.19 | -0.63 | -0.32 | -0.11 | -0.21 | 0.01 | 0.81 | 0.08 |

[1] M.R. Anders et al., in preparation.

